spond to those presented in Ref. 5 wherein it was reported that the vortex had a limiting line within which the steamlines spiraled outward from the vortex center. Conical streamlines for the same velocity field using a predictor-corrector method with a step size of $\Delta s = 0.2$ are shown in Fig. 4. It can be seen that the streamlines spiral inward to the center with a small void on the order of the step size. However, for some flow conditions, even using the predictor-corrector method with $\Delta s = 0.001$ is not sufficient to obtain the spiral trajectory.[4] For these cases, the radial velocity into the vortex center is just too small compared to the circumferential velocity.

Conclusions

The importance of step size and order of accuracy of the steamline integration is demonstrated for a model problem representing vortical flow. Results for a conical vortex show the correct expected behavior when the more accurate method is used, in contrast to erroneous results reported earlier.[5]

Acknowledgments

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Skin-Friction Measurements by Laser-Beam Interferometry

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Introduction

SOPHISTICATED methods for measuring the wall shear stress in complex turbulent flows have been developed in recent years, notably the floating-element technique,[1] the pulsed-wire technique,[2] and the oil film laser interferometer of Monson.[3] The applicability of the techniques quoted is not limited to flows with near-wall similarity of the mean velocity profile, which is required for standard techniques such as the Preston tube or Clauser chart that infer the skin friction from measured velocities. In this Note, a modified version of the laser interferometer method is reported as well as its experimental validation and application in two-dimensional, incompressible, turbulent boundary layers.

Measuring Technique

The laser interferometer technique records the changing thickness with time of a thin oil film (less than 25μm) that has been applied to the wall surface at the measuring position and experiences the shear stress of the external stream. Monson's method employs two laser beams, which are generated by an interferometer flat and focused on the wall (Fig. 1a). Each beam is partly reflected at both the surface of the oil film and the wall, thereby generating interference patterns. By means of suitable receiving optics, both reflected interfering beams are focused on photodiodes, which monitor the time change of the intensities; an example of the fringe time records is given in Fig. 1b. Each crest interval corresponds to a change in oil-film thickness by one laser wavelength.

In the present work, two modifications have been introduced into Monson's measuring system. Our experiments were performed at a much larger focal length (1 m) than the one used in Ref. 3. As a consequence, the intensity of the beams reflected from the polished aluminum plate, which was used as a wall surface, proved to be rather weak and yielded noisy fringe time records. We succeeded in improving the quality of the records (see Fig. 1b) by coating the wall with a thin transparent plastic foil sprayed black on the back side. This increased the beam intensity by a factor of three. Furthermore, we developed substantially simplified receiving optics, which replaced the more complex beam splitter of Ref. 3; we

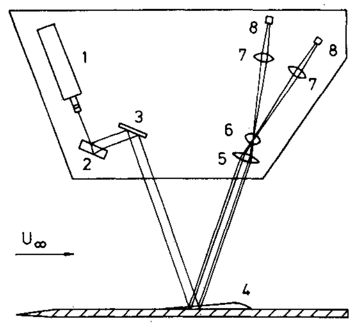

Fig. 1a Schematic of dual-laser-beam skin-friction interferometer: 1) He-Ne-laser with telescope; 2) interferometer flat; 3) mirror; 4) oil film; 5, 6, 7) biconvex lenses; 8) photodiodes.

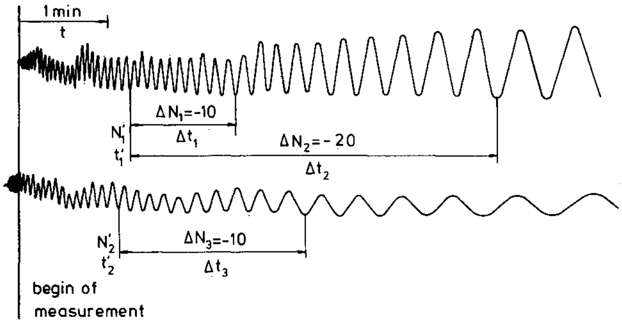

Fig. 1b Measured interferometer fringe time records, beam spacing 5 mm; upper part: downstream beam.

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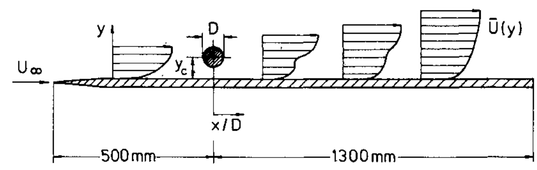

Fig. 2 Flowfield investigated (not to scale).

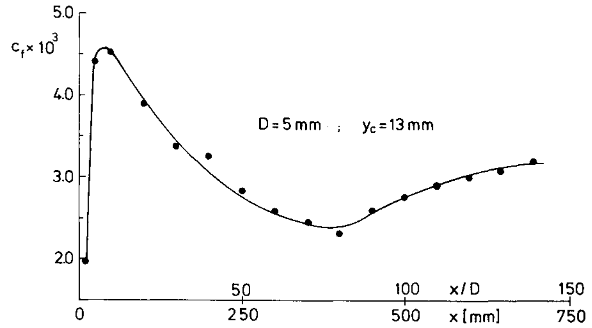

Fig. 3a Measured skin-friction distribution.

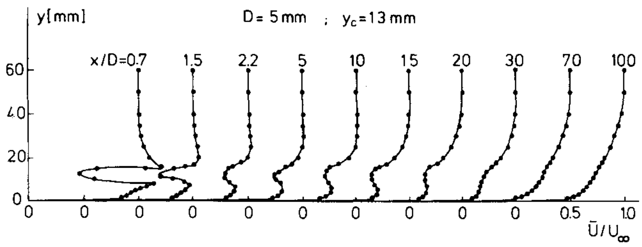

Fig. 3b Measured development of the mean velocity.

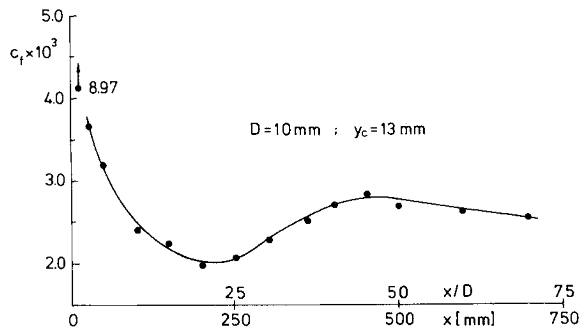

Fig. 3c Measured skin-friction distribution.

also took into account recent suggestions by Monson. A set of biconvex lenses, as sketched in Fig. 1a, separated the reflected beams and focused them on the photodiodes.

We determined the wall shear stress from the fringe time records (Fig. 1b) by means of the algebraic equation derived in Ref. 3 for the oil-film Couette flow

$$\tau = \tau(\lambda, \mu, n, \alpha, \Delta x, N_1' \cdot t_1', N_2' \cdot t_2')$$

where τ is the wall shear stress, λ is the laser wavelength, μ and n are the dynamic viscosity and index of refraction of the oil, and Δx and α are the beam spacing and angle of incidence, while $N_1' \cdot t_1'$ and $N_2' \cdot t_2'$, the products of effective fringe number and oil flow time, are computed from the measured time increments $\Delta t_{i=1,2,3}$ for prescribed fringe number increments ΔN_i.

Results

First, the modified measuring system was validated by measuring the wall shear stress distribution in an incompressible, two-dimensional, flat-plate turbulent boundary layer with a momentum-thickness Reynolds number of about 3800 and a freestream Reynolds number of 1.8×10^6; the flow was tripped at the leading edge. The experiment was carried out in the open test section (diameter 1 m, length 1.8 m) of a low-speed wind tunnel, yielding a skin-friction coefficient c_f that decreased from 0.0037 to 0.0032 between 0.5 and 1.5 m measured from the leading edge. Judging by the repeatability of the measurements, the uncertainty of the data was kept below relative errors of 7%. The major reasons for experimental scatter were traced back to temperature drift changing the oil viscosity and to dust contamination in the oil film. The results compared well with those obtained by three conventional techniques, the Preston tube, Clauser chart, and Ludwieg-Tillmann formula. The validation test showed that the modified measuring system with the simplified receiving optics produced good quality skin-friction data even at a focal length up to 1 m. Application of the instrument to other flows is straightforward, since it does not have to be calibrated.

In further measurements, attention was focused on the complex turbulent boundary layer investigated experimentally in Ref. 4 (Fig. 2). The flow was generated by the interaction of the described flat-plate boundary layer with the wake of a circular cylinder placed $y_c = 13$ mm away from the wall, a distance equal to half the thickness of the undisturbed boundary layer. In a first experiment, a cylinder of diameter $D = 5$ mm was used; in a second test, the experiment was repeated with a 10-mm-diam cylinder at the same wall distance.

The distribution of the skin friction measured by the interferometer technique for the case $D = 5$ mm (Fig. 3a) can be explained with the help of the mean velocity profiles (Fig. 3b). The velocities were measured by hot-wire anemometry; within the range $x/D < 10$ and $10 \text{ mm} < y < 20$ mm, the accuracy of the data was somewhat limited due to turbulence levels up to 30%. Acceleration of the near-wall flow within $0.7 < x/D < 10$ yielded a steep rise of the skin-friction coefficient c_f, whereas beyond $x/D = 10$ the velocity defect in the cylinder wake spread toward the wall and caused c_f to decrease. At $x/D \approx 70$, the inner-layer half-width of the velocity defect met the wall, and c_f reached its minimum value. Further downstream, flow recovery was driven by positive gradients of the Reynolds shear stress $\partial(-\overline{u'v'})/\partial y > 0$, as measured in Ref. 4, yielding increasing values of c_f. At the last measuring station, $x/D = 140$, the relaxation process had not yet reached flat-plate conditions, as indicated by the slope $\partial c_f/\partial x > 0$.

For the case $D = 10$ mm, the method succeeded in demonstrating a similar correspondence between the development of the near-wall mean velocity profiles and the measured wall shear stress distribution. The large velocity defect spread to the wall within $x/D < 20$, where a relative minimum of the skin-friction distribution was found (Fig. 3c). Beyond this position, large positive gradients of the Reynolds shear stress produced fast flow recovery within $x/D < 45$, indicated by rising values of c_f. Then the wall shear stress developed as in an undisturbed turbulent boundary layer.

In both relaxing flows, the use of standard techniques for measuring the skin friction was ruled out by the dissimilarity of the inner-layer mean velocity profiles. Indeed, test measurements by the Preston tube did not at all reproduce the measured distributions discussed. The successful development and application of the modified laser interferometer technique is currently being followed by skin-friction measurements in laminar and transitional flows.

Acknowledgment

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Erratum

Entropy Production in Nonsteady General Coordinates

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THE authors wish to thank Professor Warsi for pointing out that the parenthetical remark at the end of Ref. 6 of the above paper is incorrect. Equation (10d) of Ref. 6 is correct as it stands.